Physics 214 Final Exam Review Problems

The following questions are designed to give you some practice with concepts covered since the midterm. You should look at old practice midterms for sample problems covering the earlier course material. Some are specifically designed to be difficult in order to make sure you can go beyond simple "plug and chug" problems.

1. An electron has a wavefunction $\Psi(r,\theta,\varphi) = Cr^3e^{-r/a}$. At what radius is one most likely to find the electron?

a. $r = a$
b. $r = 2a$
c. $r = 3a$
d. $r = 4a$
e. $r = 5a$

The probability density per unit volume is $|\Psi|^2 = C^2r^6e^{-2r/a}$.
However, the probability density per unit radius must include the fact that there is more volume at large radii (because there is more surface area): $dV = 4\pi r^2 dr$. The probability density per unit radius is:

$$P(r) = 4\pi r^2|\Psi|^2 = 4\pi C^2r^8e^{-2r/a}.$$

To find the most likely radius, take the derivative and set it = 0 (I ignore $4\pi C^2$):

$$8r^7e^{-2r/a} - (2/a)r^8e^{-2r/a} = 0$$
$$8 - (2/a)r = 0$$
$$r = 4a$$

2. This electron is in what orbital angular momentum state?

a. s
b. p
c. d

$$\Psi(r,\theta,\varphi) = Cr^3 e^{-r/a}$$

There is no angular dependence, so $\ell = 0$. Non-zero ℓ requires tangential momentum, which implies a tangential "wavelength".

3. An electron in a hydrogen atom is in a p state. Which of the following statements is true?

a. The electron has a total angular momentum of $\hbar$.
b. The electron has an energy of -13.6 eV.
c. The probability to find the electron within 0.1 nm of the origin changes in time.
d. The electron's wave function has at least one node (i.e., at least one place in space where it goes to zero).
e. The electron has a z-component of angular momentum equal to $\sqrt{2}\hbar$.

a. The total angular momentum is $|L| = \sqrt{l(l+1)}\hbar$.
b. -13.6 eV is the ground state energy. Only ℓ=0 is possible there.
c. (Assuming it's an energy eigenstate): There is no time dependence.
d. $Y_{10}(\theta,\phi) \propto \cos\theta$, and $Y_{1\pm1}(\theta,\phi) \propto \sin\theta$. Both have nodes (at θ=0 or 90°).
e. The z-component must be $m\hbar$, where m is an integer.

I'm not very fond of this problem; the wording is unclear in a couple of places.

Problems 4,5, and 6 are related.

4. An electron in an infinite square well of width L = 1 nm has the wavefunction:

$$\psi(x) \propto \sqrt{\frac{2}{L}}\left[\sin(\frac{3\pi x}{L}) + \sin(\frac{5\pi x}{L}) - 2\sin(\frac{\pi x}{L})\right]$$

What is/are the possible result/results for a measurement of the electron's energy?

a. 0.376 eV
b. 2.51 eV
c. 0.376 eV, 3.39 eV, or 9.41 eV
d. 11.3 eV
e. 12.1 eV

The three terms in square brackets are the third, fifth, and first energy levels, respectively. The ground state energy is $E_1 = \frac{h^2}{8mL^2} = \frac{1.505eV}{4\times 1^2} = 0.376eV$. So the possible results are E_1, $9E_1$, and $25E_1$.

NOTE: The wave function is not properly normalized.
The coefficient in front should be $\frac{1}{\sqrt{3L}}$.

5. What is the probability of measuring the electron in the previous problem to have an energy of 0.376 eV?

$$\psi(x) \propto \sqrt{\frac{2}{L}}\left[\sin(\frac{3\pi x}{L}) + \sin(\frac{5\pi x}{L}) - 2\sin(\frac{\pi x}{L})\right]$$

a. 4
b. 0.67
c. -0.67
d. 0.5
e. 0

The probability of each result is proportional to the square of its coefficient. However, we must make sure that the probabilities sum to 1. The sums of the squares of the three coefficients inside the brackets is 1+1+4 = 6. Therefore, the probability of E_1 is 4/6. (Each of the other two results has P = 1/6.)

Note: I avoided the fact that ψ is not properly normalized.

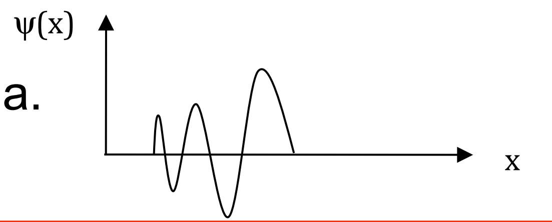
6. If indeed we measure the electron to have energy 0.376 eV, and then we shine on light of wavelength 824.5 nm, what will happen?

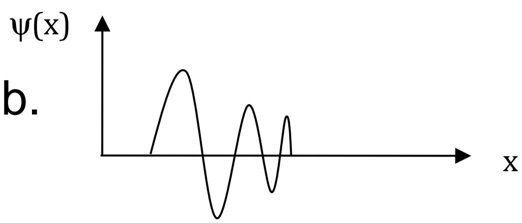
a. The electron will be excited to a state with energy 1.75 eV.
b. The electron will be excited to the state with energy 1.504 eV.
c. The electron will not be excited.

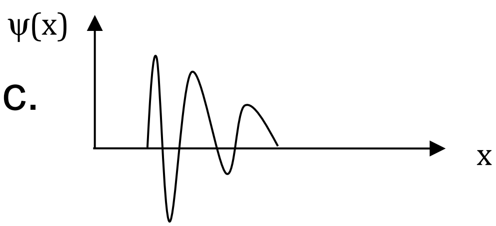
E_{photon} = 1240 ev-nm/824.5 nm = 1.504 eV .

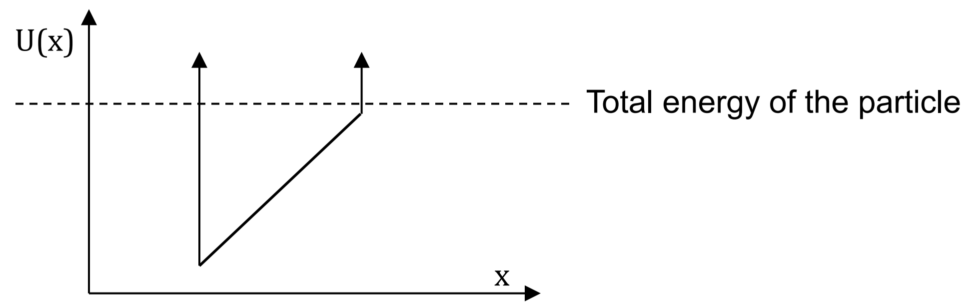
If the electron were to absorb the photon, its energy would become 0.376+1.504 = 1.88 eV.
This does not correspond to any energy level in the square well, so the electron cannot absorb the photon.

7. A particle is trapped in the potential well below.
Which of the wave functions most closely describes the particle?

a.


b.


c.


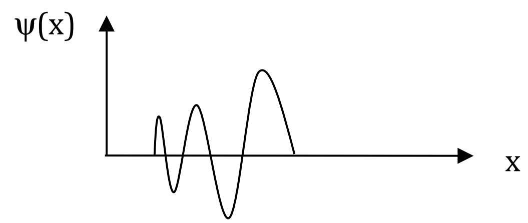


The KE is bigger on the left. Therefore:

- The wavelength is shorter.
- The amplitude is smaller.

8. What state is this particle in (where n = 1 is the ground state)?

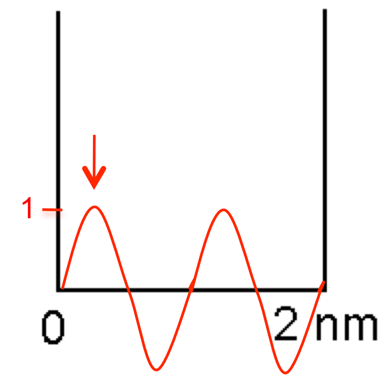
a. n = 2
b. n = 3
c. n = 4
d. n = 5
e. n = 6



Four nodes means n = 5.

9. An electron is in the 3rd excited state of a 2-nm wide infinite square well. What is the probability of measuring the electron to be between x = 0.23 nm and x = 0.27 nm?

a. 0.04
b. 0.10
c. 0.16
d. 0.32
e. 0.64



Third excited state means n = 4. The normalized wave function is: $\psi_4(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{4\pi x}{L}\right)$
Δx (= 0.04 nm) is so small that the probability can be approximated by:

$$P \sim |\psi(0.25)|^2 \Delta x = 0.04 \sin^2(\pi/2) = 0.04$$

10. An electron with total energy E approaches a barrier of height U_0 and width L. Assuming $E<U_0$, which one of the following changes will **increase** the probability for the electron to appear on the other side of the barrier?

a. increase L
b. increase E
c. increase U_0

$T \propto e^{-2KL}$, and $K \propto \sqrt{U_0 - E}$

Therefore:

a. Increasing L decreases T.

a. Increasing E decreases K, which increases T.

a. Increasing U_o increases K, which decreases T.

11. Which of the following normalized wave functions for the infinite square well has the shortest period of oscillation in time?

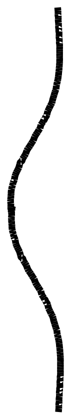
a. $(\sin(\pi x/L) + \sin(2\pi x/L)) / \mathrm{sqrt}(L)$
b. $(\sin(2\pi x/L) + \sin(3\pi x/L)) / \mathrm{sqrt}(L)$
c. $(\sin(\pi x/L) + \sin(3\pi x/L)) / \mathrm{sqrt}(L)$

These are superpositions of energy eigenstates: E_1-E_2, E_2-E_3, and E_1-E_3 respectively. Each oscillates at the beat frequency, $f = \Delta E/h$.
The shortest period means the highest frequency, or largest ΔE.

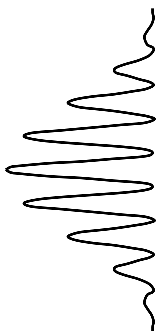
Problems 12 and 13 are related.

12. Which of the following probability distributions will you observe from a beam of electrons passing through a double slit with one slit covered? (Assume that the detection screen is far away from the slits, i.e, the diagrams are not drawn to scale)?

a.



b.



With one slit covered, you simply get the single slit diffraction pattern.

13. Now both slits are unblocked. However, we modify the experiment in the following way: We prepare the electrons incident on the slits so that they all have their spins "pointing up", i.e., so that $m_s = +1/2$. We install a tiny radio-coil near the top slit (this is only a thought experiment!), so that the spin of any electron that passes through the top slit is flipped (without affecting the spin of electron passing through the bottom slit). Now which pattern do we see?

a.
b.

The electron spin contains information about which slit the electron went through. That destroys the interference pattern. So, we still see a single slit diffraction pattern.

14. What frequency of electromagnetic radiation will flip a "spin up" electron to a "spin down" electron in a magnetic field of 2.0 T?

a. $2.4*10^9$ Hz
b. $4.1*10^9$ Hz
c. $5.6*10^{10}$ Hz
d. $7.1*10^{11}$ Hz
e. $8.8*10^{12}$ Hz

The photon energy must match the difference in energy levels.

$\Delta E = 2\mu B = 2(9.28\times10^{-24}\ \text{J/T})(2\ \text{T}) = 3.72\times10^{-23}\ \text{J}$

$f = \Delta E/h = 3.72\times10^{-23}\ \text{J} / 6.63\times10^{-34}\ \text{J-s} = 5.61\times10^{10}\ \text{Hz}$

15. A photon has energy 3 eV. What is its momentum?

a. 0
b. $1.6*10^{-27}$ kg m/s
c. $9.4*10^{-34}$ kg m/s

For photons, $p = E/c = (3\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})/(3\times10^{8}\ \text{m/s}) = 1.6\times10^{-27}$ SI units.

Be careful to use the correct relation between p and E.

16. A laser with wavelength 300 nm illuminates a metal in a photoelectric effect experiment. It takes a stopping potential of 2 Volts to halt the ejected electrons. What is the work function of the metal?

a. 1.0 eV
b. 2.1 eV
c. 3.2 eV

The work function (the amount of energy an electron loses while escaping from the metal) is the photon energy minus the energy the electrons emerge with:

$$\Phi = E_{photon} - eV_{stop}. = 1240 \text{ eV}\cdot\text{nm}/300 \text{ nm} - 2 \text{ eV} = 2.13 \text{ eV},$$

Problems 17 and 18 are related.

17. An electron is confined to a rectangular region in space with sides L_x =2 nm, L_y = 3 nm, L_z = 2 nm. What is the energy of the ground state?

a. 0.094 eV
b. 0.19 eV
c. 0.23 eV

$$E_{n_x n_y n_z} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right)$$

3-D rectangular well:

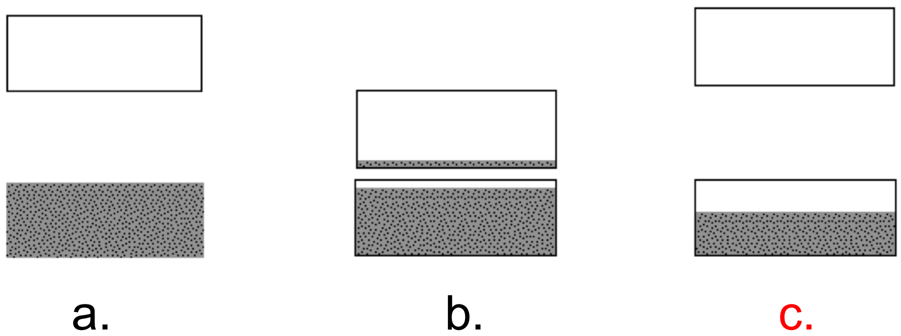
Ground state: (n_x,n_y,n_z) = (1,1,1), so, E = 0.230 eV

18. What is the degeneracy of the 1st excited state for the electron in the previous problem (neglecting the effect of spin)?

a. 1
b. 2
c. 3

$L_x = L_z < L_y$, so it costs less energy to make $n_y = 2$ than to make either n_x or $n_z = 2$. The first excited state is (1,2,1). No degeneracy. The states (2,1,1) and (1,1,2) are degenerate, but they are the 2nd excited state.

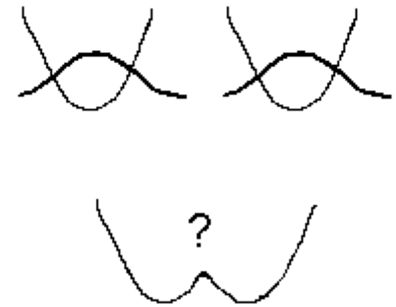
19. Which of the following energy band pictures corresponds to a conductor?

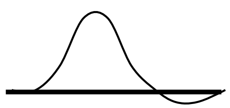


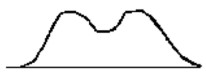
In a conductor, the electrons must have unoccupied energy states nearby, or it requires too much energy to get them moving. So, a is ruled out. in b, a few electrons have nearby energy states, but not many (and this is only due to thermal excitation). It is a semiconductor. In picture c, there are nearby states even at absolute zero temperature. That is the conductor.

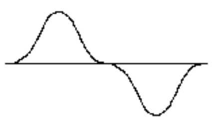
Problems 20 and 21 are related.

20. Two harmonic oscillators in their ground states are brought near each other. Which of the following pictures shows the correct 1st excited state for the combined system?



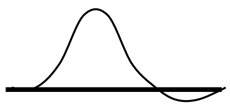
a. 

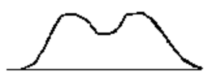
b. 

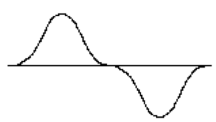
c. 

This is very similar to the problem with two square wells with a narrow barrier between them. We can immediately rule out a, because the energy eigenstates of symmetric potentials are always even or odd functions. Of b and c, b has lower energy, because it is more spread out (longer wavelength). So, c is the first excited state.

21. Assume there is one electron from each harmonic oscillator (and neglect electrostatic interactions between the electrons). If the “molecule” is in its lowest energy state, one of the electrons is in state (b.) above. Which of the above pictures is appropriate for the wave function of the second electron?

a. 

b. 

c. 

The second electron must be in a different quantum state. However, this is achieved by putting it in the other available spin state, because it does not cost energy to do that.

22. A beam of electrons is sent toward a potential barrier (height = 2 eV) with velocity $6x10^5$ m/s. If 97.5% of the incident beam is reflected, what is the width of the barrier?

a. 0.01 nm
b. 0.05 nm
c. 0.1 nm
d. 0.5 nm
e. 1 nm

$T = 0.025 = Ge^{-2KL}$. We need to know the electrons' energy in eV.

$E = ½mv^2 = 0.5(9.11×10^{-31})(6×10^5)^2/1.60×10^{-19} = 1.02$ eV

So, G = 4 and $K = 2π√((U-E)/1.505) = 5.12\ nm^{-1}$

Thus: $L= -ln(0.025/4)/(2×5.12\ nm^{-1}) = 0.50$ nm.

Problems 23 and 24 are related.

23. A hydrogen atom in its ground state traveling in the +x-direction is passed along the through a Stern-Gerlach apparatus, producing a set of peaks. The uppermost peak only is then passed through *another* Stern-Gerlach apparatus (with the same magnetic field gradient dB/dz as the first). How many peaks are observed in the output of the second Stern-Gerlach apparatus?

a. 0
b. 1
c. 2
d. 3
e. 4

Once you measure a quantity, the wave function “collapses” to correspond to the measured result. Therefore, unless we do something to the atom to change it’s magnetic moment, a second measurement will obtain the same answer as the first.

24. If instead we were to rotate the second Stern-Gerlach apparatus by 90°, so that the gradient was dB/dy instead, now how many peaks would be observed?

a. 0
b. 1
c. 2
d. 3
e. 4

There is no QM state corresponding to definite values of both s_x and s_y. Therefore, after we measure s_x (and therefore, ψ is a state of definite s_x), it must be a superposition of various s_y states. The electron has spin ½, so there are two of them.

We didn't spend much time on this topic , so it's not a likely exam question.

25. What are the quantum numbers n and l of the outermost electron of a Br atom? Br has 35 electrons.

a. n=3, l=0
b. n=3, l=1
c. n=4, l=0
d. n=4, l=1
e. n=4, l=2

The filling order (and number of electrons) is:

1s (2)
2s (2)
2p (6)
3s (2)
3p (6)
4s (2)
3d (10)
4p (6) That gets us to 36, so the answer is 4p (n=4,l=1).

26. If the outermost electron is now excited (e.g., by a collision) to the n = 5, l = 1 state, to which final state(s) could the electron fall back down by emitting a photon?

a. n=4, l=3
b. n=4, l=2
c. n=5, l=0
d. n=4, l=1
e. n=3, l=2

The electron is now in the 5p state. There are lower energy unoccupied states in the 4d (4,2), 5s (5,0), and 4p (4,1) orbitals. Transition to 4p violates the $\Delta l = \pm 1$ selection rule, so there are two correct answers.

Problems 27-29 refer to this situation:

A calcium ion (charge |e|, mass = 6.65×10^{-26} kg) is trapped in an electromagnetic potential that approximates a **harmonic oscillator**. The frequency associated with the oscillation of the ion in the trap is 100 kHz.

27. If one wanted to excite the ion from the ground state of the trap *directly* to the *second excited state*, one might shine on radio waves with frequency:

a. 100 kHz
b. 200 kHz
c. 800 kHz

The energy spacing in a harmonic oscillator is $\Delta E = hf$. This is also the energy of a photon with the same frequency. So, the frequency photon that is needed to jump $2\Delta E$ is $2f = 200$ kHz.

28. At time t = 0, the ion is prepared into an equal superposition of the ground state and the *second* excited state, $\psi = \frac{1}{\sqrt{2}}(\psi_0 + \psi_2)$. Which of the following describes the likely location of the ion:

a. The ion is more likely to be found in the left-hand side of the trap.
b. The ion is more likely to be found in the right-hand side of the trap.
c. The ion is equally likely to be found in either half of the trap.

The potential is symmetric, so all of the energy states are either even or odd (symmetric or antisymmetric) functions of position. The symmetry alternates as we go up the energy ladder: even, odd, even, odd, ...

ψ_0 and ψ_2 are both even functions, so their sum is even as well. This means that the probability on the left equals the probability on the right.

29. We now let the system evolve in time. Which of the following best describes the future behavior of the ion:

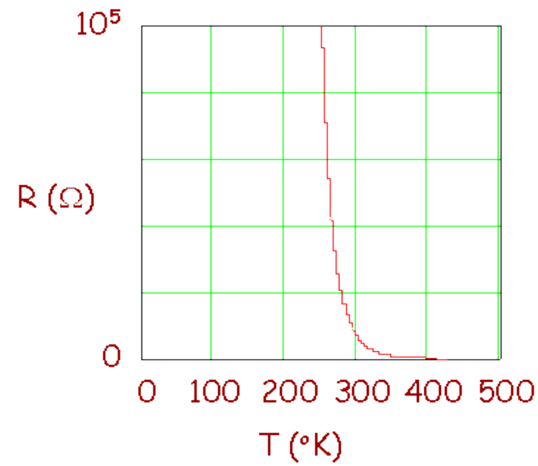
a. The ion will “slosh” back and forth from the left-hand side of the well to the right-hand side.
b. The ion will “slosh” back and forth from being mostly located near the center of the well to being mostly located away from the center (i.e., nearer the “edges” of the well).
c. The probability density of the ion will not change over time.

The two states have different energy, and therefore different frequencies. Therefore the interference (the phase difference) will change with time (c is not correct). No matter what the phase difference is, the superposition of ψ_0 and ψ_2 remains an even function. Therefore the probability is always symmetric (a is not correct).

30. Consider the following curve of resistance versus temperature.

What kind of material is this?

a. insulator
b. semiconductor
c. Metal



The resistance is not large, so it's not an insulator. The resistance decreases with temperature, so it's a semiconductor, not a metal.